

A THEORY OF EMPLOYMENT STRUCTURE*

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I. *Introduction*

The purpose of this paper is to examine the degree of employment internalization in a fairly general framework. The model we are going to discuss has the following characteristics. First, we consider the existence of productive, firm-specific human capital as the primary cause of employment internalization. Secondly, we feel that there is a very strong interaction between dismissal probability and workers' investment in specific or general human capital. Thirdly, we understand, therefore, that each firm's employment policy or institution which is linked with the dismissal probability is a device that encourages workers to (or not to) invest in specific human capital.

A model with the same characteristics has already been developed in Arai (1988). However, the model described in this paper assumes a slightly more general distribution of the product price. This generalization might appear trivial at first sight, but it will eventually enable us to show the entire structure of different types of employment characterized by different degrees of employment internalization. The computation for this generalization is much more complex than that for the original model. There are also a few improvements or clarifications in exposition in this paper. Before entering into the discussion of the model, we would like to set forth the basic motivation and significance of this paper.

We consider a worker of a firm who faces a decision as to investment in firm-specific human capital (specific capital hereafter) and/or general human capital (general capital hereafter). Our basic premise is that the portion of time and/or effort a worker invests in specific capital, as opposed to general capital, is greater the more complete is his job security or the more useful is the specific capital in other firms,¹ but if the firm guarantees job security and a sufficiently high wage, it may suffer a loss when the product price becomes low. On the other hand, if much productive specific capital is accumulated, the probability

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¹ Some examples illustrate this point. An extreme example is the case in which a firm requests its workers to learn a minor foreign language, because their productivity is likely to increase substantially if they can understand it when they are transferred to the country where it is spoken. Acquiring knowledge of the peculiarities of the machines workers operate [Doeringer and Piore (1971)], of the personalities of their co-workers, or of the system and values of their firms raises the same question. In general, we can regard the completion of any training (either formal or informal) or work which will later bring about a difference in the usefulness of this experience between inside and outside the firm as accumulation of specific capital. Thus we treat very common or even daily problems of workers and firms in this paper.

of dismissal will be low. By making the worker's investment decision endogenous, we can see in our model how much of the two kinds of human capital will actually be accumulated in this game situation.²

In our game we will consider the following three mutually exclusive but exhaustive types of employment conditions: lifetime employment equilibrium (LEE), where the worker is never dismissed and accumulates some amount of specific capital; conditional employment equilibrium (CEE), where the probabilities of dismissal and retention are both positive, but where there is some accumulation of specific capital; and spot-market employment equilibrium (SEE), where the worker accumulates only general capital or is certain to be dismissed. CEE can be divided further based on dismissal-retention criteria, the detail of which will be discussed later. The equilibrium that involves only general capital investment is called SEE because in this case there is no need for a long-term labor contract in our model. We will demonstrate under what circumstances each of these types of employment arises.

The significance of the model is that it provides a theory on a fundamental mechanism through which internal labor markets arise. Its advantage is that it can produce a spectrum of different types of employment which differ in the degree of employment internalization. The most important feature of the model in this paper is that the assumption of a slightly more general distribution of the product price enables us to conjecture the entire employment structure in more general cases. The model in Arai (1988) does not have this feature. There are some models which show explicitly or implicitly why internal labor markets arise. However, none of them can show why some firms have them and others do not.³

We shall build the model in Section II and obtain the equilibrium in Section III. In Section IV we shall discuss some characteristics of the equilibrium. In Section V we shall conclude the paper with a short summary and some additional remarks.

II. *The Model*

We consider a commonly used two-period model of a worker-firm game. In the first period human capital investment is undertaken, and in the second period production is carried out using the human capital accumulated in the first period. At the beginning of the first period the two parties strike an employment contract. The contract reflects the firm's strategy and has essentially two components. One is the wage for the second period and the other is the level of job security, which will be discussed in detail later. Taking the firm's strategy into consideration, the worker decides the allocation of his total resources, such as time and effort, toward either specific or general capital formation. Thus the worker's strategy has only one component, i.e., the portion of his total resources invested in specific capital.

² Some authors have analyzed the problem of labor turnover in relation to specific and general capital. But they have assumed either that the absolute amounts of the two kinds of capital are fixed [Mortensen (1978)], that the ratio of the amounts of the two kinds of capital is fixed [Pencavel (1972)], or that only specific training can be undertaken on the job [Donaldson and Eaton (1976) and Hashimoto (1979)].

³ See Coase (1937), Oi (1962), and Azariadis (1975). Williamson (1985, Chap. 10) pursues the same goal as the one in this paper, but his work is sketchy and incomplete.

When the employment contract is offered, the price of the product is uncertain. But since this uncertainty diminishes gradually, we assume that the price will be known for certain at the end of the first period. As soon as price information is obtained, the previously offered contract comes into effect. If the firm dismisses the worker, he has to work outside the firm using the general and specific capital he accumulated in the first period. If not, he can continue to work for the same firm with the same capital portfolio. Of course, he is free to quit at the end of the first period and work for a new firm in the second period.

Suppose the worker has a fixed amount of resources that can be invested in human capital in the first period. These resources are either time or effort or both, but to simplify the exposition let a fixed amount $c > 0$ of his time represent the total resources available for investment. There can be slightly different interpretations for this. One is that c represents the worker's total portal-to-portal hours in the first period and he never engages in production in this period. The first period is simply an investment period. This is not so extreme an interpretation as we will see below, and assumptions like this are sometimes made in economics. Another interpretation is that c represents his total portal-to-portal hours in the first period as above but he engages in production from the beginning to the end of the period. Human capital investment in the first period is undertaken as on-the-job training or learning by doing. This is a reasonable interpretation and fits our model nicely. Still another interpretation is that c represents a certain portion of his total portal-to-portal hours in the first period and he never engages in production in this portion of the first period. During the remaining portion of the first period, he engages only in production and no training is undertaken. This interpretation is fairly realistic, but it is essentially the same as the first one.

Choosing units suitably, let one unit of his time produce one unit of specific capital or one unit of general capital. The amount of specific capital he accumulates is denoted by x and that of general capital by y , where $x \geq 0$, $y \geq 0$ and $x + y = c$. This x represents the worker's only strategy in our game. We assume that there is some difference in productivity between the two kinds of human capital. More specifically, let α units of general capital be equivalent in productivity to one unit of specific capital within the firm. If $\alpha \leq 1$, there is little to analyze in our framework. Thus we assume $\alpha > 1$ in the following unless we explicitly assume the contrary.

Again choosing a suitable unit, let one unit of general capital have a capacity for producing one unit of output in the firm in the second period. Thus the total physical product of the worker is equal to $\alpha x + y$, if the two parties do not separate. The price of the product in the second period is denoted by p . We assume in our model that p can take on three values,⁴ i.e., p_1 , p_2 , and p_3 , where $0 < p_1 < p_2 < p_3$. The probability that $p = p_i$ is $\pi_i > 0$ ($i = 1, 2, 3$), where $\sum \pi_i = 1$. The worker's value product can be written as $p(\alpha x + y)$. He will receive a fixed wage w in the second period if the two parties do not separate. This w is one of the two components in the firm's strategy.

If the worker is dismissed after the price information is obtained, he has to work in the second period for a new firm using the human capital portfolio (x, y) he accumulated in the first period. The specific capital now is not as productive in the new firm as in the

⁴ It is mainly this assumption that differs from the model in Arai (1988). We assumed there that p can take on only two values.

old one. Let one unit of the specific capital be equivalent in productivity to β units of general capital, where $0 \leq \beta < \alpha$. β is a measure of the specificity of specific capital. The smaller the value of β , the higher the specificity. We assume that the wage the worker will receive in the new firm is equal to $r(\beta x + y)$, where $r > 0$ is fixed, i.e., the wage is nonrandom and proportional to his physical productivity there. This assumption approximates the case where there is a large market for general capital. Moreover, it simplifies the worker's decision to quit. He will quit if and only if the initial firm offers a lower wage than the highest possible wage he can earn elsewhere. We note that the worker seeks to accumulate specific capital unconditionally if $\beta \geq 1$. In this case specific capital is more useful than general capital even outside the initial firm. Though we cannot exclude this exceptional case *a priori*, we are interested mainly in the case where $\beta < 1$.

Now we will consider again the initial firm-worker relationship and see how the firm decides to dismiss or retain the worker. We assume that the firm dismisses the worker if and only if $p(\alpha x + y) + t < w$. This t measures the job security the firm offers, and *ceteris paribus* the larger the value of t , the more unlikely a dismissal. If $t > 0$, it is possible that the firm will retain the worker even when his value product is less than his wage. This is the case of labor hoarding. If $t < 0$, the firm might dismiss him even when his value product is larger than his wage. This t is the second component in the firm's strategy, and we call it the security limit of employment. Using the firm's dismissal-retention rule, we can show the probabilities of dismissal and retention as functions of the combination (w, t, x) of the strategies of the two parties. Making use of the constraint $x + y = c$, we have: probability of dismissal = $\text{Prob}\{p((\alpha - 1)x + c) + t < w\}$ and probability of retention = $\text{Prob}\{p((\alpha - 1)x + c) + t \geq w\}$.

Given (w, t) , we can write the worker's evaluation $U(x)$ of the firm's strategy as a function of his choice of x . Suppose that the worker is interested in maximizing the expected wage he will receive. Then the evaluation function represents the expected wage he will receive if he chooses x in reaction to the firm's strategy (w, t) . Considering the above dismissal-retention rule, for $0 \leq x \leq c$ we have:

$$(1a) \quad U(x) = w, \quad \text{if } \frac{w-t}{p_1(\alpha-1)} - \frac{c}{\alpha-1} \leq x;$$

$$(1b1) \quad U(x) = \pi_1 r(c - (1-\beta)x) + (\pi_2 + \pi_3)w, \\ \text{if } \frac{w-t}{p_2(\alpha-1)} - \frac{c}{\alpha-1} \leq x < \frac{w-t}{p_1(\alpha-1)} - \frac{c}{\alpha-1};$$

$$(1b2) \quad U(x) = (\pi_1 + \pi_2)r(c - (1-\beta)x) + \pi_3 w, \\ \text{if } \frac{w-t}{p_3(\alpha-1)} - \frac{c}{\alpha-1} \leq x < \frac{w-t}{p_2(\alpha-1)} - \frac{c}{\alpha-1};$$

$$(1c) \quad U(x) = r(c - (1-\beta)x), \quad \text{if } x < \frac{w-t}{p_3(\alpha-1)} - \frac{c}{\alpha-1}.$$

If x is in the region of (1b1) for example, the probability of retention is $(\pi_2 + \pi_3)$ and that of dismissal is π_1 . Since the worker will work either for the initial firm or for a new one in

the second period, taking his evaluation function into account enables us to write his expected wage $S(x)$ as

$$(2) \quad S(x) = \max \{U(x), r(c - (1 - \beta)x)\}, \quad 0 \leq x \leq c.$$

The second component in the braces represents the wage he will get if he quits with x units of specific capital. Given (w, t) by the firm, he chooses x so as to maximize $S(x)$ subject to $0 \leq x \leq c$.

Next we consider the firm's payoff. We assume that if the firm dismisses the worker, it can instantaneously hire a new worker with c units of general capital for the market wage rate.⁵ Let v_i denote the profit it gains if it dismisses the initial partner when $p = p_i$. Then

$$(3) \quad v_i = \begin{cases} (p_i - r)c & \text{if } r < p_i, \\ 0 & \text{if } p_i \leq r, \end{cases} \quad (i=1, 2, 3).$$

Using (3), we can write the firm's expected profit $V(w, t)$ as follows:

$$(4a) \quad V(w, t) = E(p)((\alpha - 1)x + c) - w;$$

$$(4b1) \quad V(w, t) = \pi_1 v_1 + \pi_2 [p_2((\alpha - 1)x + c) - w] + \pi_3 [p_3((\alpha - 1)x + c) - w];$$

$$(4b2) \quad V(w, t) = \pi_1 v_1 + \pi_2 v_2 + \pi_3 [p_3((\alpha - 1)x + c) - w];$$

$$(4c) \quad V(w, t) = \pi_1 v_1 + \pi_2 v_2 + \pi_3 v_3;$$

where $E(p) = \sum \pi_i p_i$. The region of (w, t, x) for each of the above four equations is the same as the corresponding region in (1). In order for (4a), (4b1), and (4b2) to hold, it is necessary that $U(x) \geq rc$ if $\beta < 1$ or that $U(x) \geq \beta rc$ if $\beta \geq 1$, otherwise the firm's payoff is equal to that in (4c). In the region for (4c), the above condition is automatically satisfied with an equality. If $\beta < 1$, the worker's highest possible alternative wage in the second period is equal to rc which he can achieve by choosing $x=0$. If the probability that the firm will employ him in the second period is positive, the expected wage for the worker must be at least equal to rc . If $\beta \geq 1$, his highest possible alternative wage in the second period is equal to βrc which he can achieve by choosing $x=c$. The firm chooses (w, t) so as to maximize $V(w, t)$ subject to $U(x) \geq rc$ if $\beta < 1$ or subject to $U(x) \geq \beta rc$ if $\beta \geq 1$ in anticipation of the worker's reaction to the chosen strategy.

We would like to add notes here. There are cases where the worker chooses $x=0$ and the firm dismisses him formally as in (1c) with $\beta < 1$. But we can assume the firm rehires the worker for rc as his wage, if it intends to employ a worker with only general capital after it dismisses the old one. The reason is that the firm is equally served by a new worker or the old one when both have only general capital, and the old worker is completely indifferent to a new firm and the old one because the wage is equal to rc in both. This can arise because the two parties have no special attachment for each other due to the lack of specific capital investment.

⁵ We will see in the process of our discussion that this assumption is necessary only for formality.

III. *Equilibrium Types of Employment*

In this section we consider the equilibrium of the game presented in the previous sections. As we mentioned in Section I, we divide CEE into two cases: one is called CEE1 where the firm dismisses the worker only when $p=p_1$, and the other is the case called CEE2 where it dismisses him only when $p \leq p_2$. We are interested mainly in the conditions in which LEE, CEE1, CEE2, or SEE arises. We will find these conditions by first characterizing each equilibrium and then comparing the firm's payoffs. Since the case where $\beta \geq 1$ is quite different from that where $\beta < 1$, we treat them separately.

We begin by characterizing each equilibrium when $\beta < 1$. Consider LEE first. As mentioned before, it is necessary that $w \geq rc$. (1a) implies that the worker chooses $x \geq (w-t)/p_1(\alpha-1) - c/(\alpha-1)$. Because his wage is independent of the chosen level of x , we assume for simplicity that he chooses $x = (w-t)/p_1(\alpha-1) - c/(\alpha-1)$. We can easily show that this does not lose generality. Since $0 < x \leq c$ in LEE, $t < w - p_1c$ and $t \geq w - \alpha p_1c$. Substituting the above x level into (4a), we have $V(w, t) = (E(p)/p_1 - 1)w - E(p)t/p_1$. The firm maximizes this $V(w, t)$ subject to $w \geq rc$, $t < w - p_1c$, and $t \geq w - \alpha p_1c$. The equilibrium is obviously $(w, t, x) = (rc, (r - \alpha p_1)c, c)$ and the firm's payoff V^L in this equilibrium is given by

$$(5) \quad V^L = (\alpha E(p) - r)c.$$

Next we consider SEE. Any of the firm's strategies that induce the worker to choose $x=0$ leads to SEE. There are a continuum of such strategies.⁶ The firm's payoff V^S in this equilibrium is the same as (4c), i.e.,

$$(6) \quad V^S = \pi_1 v_1 + \pi_2 v_2 + \pi_3 v_3.$$

According to (1b1), CEE1 can arise only when the worker chooses $x = (w-t)/p_2(\alpha-1) - c/(\alpha-1)$. By definition and assumption $0 < x \leq c$, i.e., $t < w - p_2c$ and $t \geq w - \alpha p_2c$. Substituting the above x level into (1b1), we can write the condition $U(x) \geq rc$ as

$$(7) \quad t \geq \left[1 - \frac{\pi_2 + \pi_3}{\pi_1} \frac{\alpha - 1}{1 - \beta} \frac{p_2}{r} \right] w + \frac{(\pi_2 + \pi_3)\alpha + \pi_1\beta - 1}{\pi_1(1 - \beta)} p_2c.$$

Substituting again the above x level into (4b1), we can write the firm's expected profit as $V(w, t) = \pi_1 v_1 - (\pi_2 + \pi_3 p_3/p_2)t + \pi_3(p_3/p_2 - 1)w$. In CEE1 the firm maximizes this $V(w, t)$ subject to the three inequality conditions we have just obtained. A simple computation reveals that CEE1 is $(w, t, x) = [(1 - \pi_1\beta)rc/(\pi_2 + \pi_3), (1 - \pi_1\beta)rc/(\pi_2 + \pi_3) - \alpha p_2c, c]$, that the firm's maximized expected profit V^{C1} in this equilibrium is given by

$$(8) \quad V^{C1} = \pi_1 v_1 + (\pi_2 p_2 + \pi_3 p_3)\alpha c - (1 - \pi_1\beta)rc,$$

and that these hold only when⁷

⁶ Two examples are $(w, t) = (3p_3\alpha c, p_3\alpha c)$ and $(w, t) = (2p_3\alpha c, 0)$.

⁷ (9) arises from the condition that the slope of the iso-expected profit lines must be larger than or equal to that of the boundary of (7) in the (w, t) plane. If this is not satisfied, CEE1 does not exist. The equilibrium is not unique when an equality holds in (9). In particular x can be less than c . We ignore the detail of this special case for simplicity of exposition. A similar note applies to CEE2.

$$(9) \quad \alpha \geq \frac{\pi_1(1-\beta)}{\pi_2 p_2 + \pi_3 p_3} r + 1.$$

CEE2 can be obtained similarly. We can show that CEE2 is $(w, t, x) = [(1 - (\pi_1 + \pi_2)\beta)rc/\pi_3, (1 - (\pi_1 + \pi_2)\beta)rc/\pi_3 - \alpha p_3 c, c]$, that the firm's maximized expected profit V^{c2} is given by

$$(10) \quad V^{c2} = \pi_1 v_1 + \pi_2 v_2 + \pi_3 \alpha p_3 c - (1 - (\pi_1 + \pi_2)\beta)rc,$$

and that these hold only when

$$(11) \quad \alpha \geq \frac{(\pi_1 + \pi_2)(1-\beta)}{\pi_3 p_3} r + 1.$$

We can show which type of equilibrium will actually arise by comparing the firm's payoffs expressed in (5), (6), (8), and (10). Table 1 summarizes the relative magnitude of the four payoffs in the four regions of r . For instance, when $p_1 \leq r < p_2$, $V^L \geq V^{c1}$ if and only if (iff) $\alpha \geq \beta r/p_1$. The five blanks imply that the corresponding relationships hold without constraints. The attached numbers imply the equality conditions.⁸ We prove only one relationship here. All the others can be proved similarly. Assume $p_1 \leq r < p_2$ and consider the relationship between V^{c1} and V^S . $V^{c1} - V^S = (\pi_2 p_2 + \pi_3 p_3)\alpha c - (1 - \pi_1 \beta)rc - \pi_2(p_2 - r)c - \pi_3(p_3 - r)c = [(\pi_2 p_2 + \pi_3 p_3)\alpha - \pi_1(1 - \beta)r - \pi_2 p_2 - \pi_3 p_3]c$. Therefore, $V^{c1} \geq V^S$ iff $\alpha \geq \pi_1(1 - \beta)r/(\pi_2 p_2 + \pi_3 p_3) + 1$.

Since the equilibrium is the firm's strategy (w, t) and the corresponding strategy x chosen by the worker that maximize the expected profit, in order to see which type of equilibrium will actually arise we have only to examine which of the four payoffs of the firm is the largest in each of all the possible cases that are classified according to different combinations of the values of the exogenous parameters. Using Table 1, we can show which payoff is the largest in (r, α) planes. The combinations of the values of the exogenous parameters that give rise to different types of equilibrium can be shown as areas in (r, α) planes with the values of parameters other than r and α determining the boundaries of the areas.

Since we have already seen the relative magnitude of the four payoffs, our remaining task is simply to classify the combinations of different values of the exogenous parameters in (r, α) planes and then to find which payoff is the largest in each of the classified areas. However, the actual computation is complex, since there are a number of cases to be considered. Therefore, I write this part in the Appendix.⁹

We summarize the results here. To do so it is convenient to divide cases according to the magnitude of β . The critical values of β are those we see in the Appendix, i.e., $\beta = p_1/E(p)$, $\beta = p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p)$, and $\beta = p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3)$. Though the first critical value is smaller than the others, the relative magnitude between the latter two cannot be determined *a priori*. So we first assume that $p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p) < p_2/(\pi_1 p_2 + \pi_2 p_2 +$

⁸ We adopt simplification of notations in Table 1. The statement that $A \geq B$ iff $C \geq D$ implies that $A > B$ iff $C > D$, that $A = B$ iff $C = D$, and that $A < B$ iff $C < D$. Only for the five blanks in the table do the relationships between payoffs hold with strict inequalities. As stated in the text, (2-1), (2-2), etc. imply the corresponding equality conditions. For example, (2-1) is equivalent to $\alpha = \beta r/p_1$.

⁹ Before reading the Appendix, the readers are recommended to read the summary of the results written in the paragraph immediately below and to look at Figures 1 through 5. The Appendix will be more easily understood if the figures are referred to in the discussion.

TABLE 1

	$r < p_1$	$p_1 \leq r < p_2$	$p_2 \leq r < p_3$	$p_3 \leq r$
$VL \geq VCI$		(2-1) $\alpha \geq \frac{\beta}{p_1} r$	(3-1) $\alpha \geq \frac{\beta}{p_1} r$	(4-1) $\alpha \geq \frac{\beta}{p_1} r$
$VL \geq VS$		(2-2) $\alpha \geq \frac{\pi_1}{E(p)} r + \frac{\pi_2 p_2 + \pi_3 p_3}{E(p)}$	(3-2) $\alpha \geq \frac{\pi_1 + \pi_2}{E(p)} r + \frac{\pi_3 p_3}{E(p)}$	(4-2) $\alpha \geq \frac{1}{E(p)} r$
$VCI \geq VS$	(1-3) $\alpha \geq \frac{\pi_1(1-\beta)}{\pi_2 p_2 + \pi_3 p_3} r + 1$	(2-3) $\alpha \geq \frac{\pi_1(1-\beta)}{\pi_2 p_2 + \pi_3 p_3} r + 1$	(3-3) $\alpha \geq \frac{\pi_1(1-\beta) + \pi_2}{\pi_2 p_2 + \pi_3 p_3} r + \frac{\pi_3 p_3}{\pi_2 p_2 + \pi_3 p_3}$	(4-3) $\alpha \geq \frac{1 - \pi_1 \beta}{\pi_2 p_2 + \pi_3 p_3} r$
$VCI \geq VC2$			(3-4) $\alpha \geq \frac{\beta}{p_2} r$	(4-4) $\alpha \geq \frac{\beta}{p_2} r$
$VL \geq VC2$		(2-5) $\alpha \geq \frac{(\pi_1 + \pi_2)\beta - \pi_2}{\pi_1 p_1 + \pi_2 p_2} r + \frac{\pi_3 p_3}{\pi_1 p_1 + \pi_2 p_2}$	(3-5) $\alpha \geq \frac{(\pi_1 + \pi_2)\beta}{\pi_1 p_1 + \pi_2 p_2} r$	(4-5) $\alpha \geq \frac{(\pi_1 + \pi_2)\beta}{\pi_1 p_1 + \pi_2 p_2} r$
$VC2 \geq VS$	(1-6) $\alpha \geq \frac{(\pi_1 + \pi_2)(1-\beta)}{\pi_3 p_3} r + 1$	(2-6) $\alpha \geq \frac{(\pi_1 + \pi_2)(1-\beta)}{\pi_3 p_3} r + 1$	(3-6) $\alpha \geq \frac{(\pi_1 + \pi_2)(1-\beta)}{\pi_3 p_3} r + 1$	(4-6) $\alpha \geq \frac{1 - (\pi_1 + \pi_2)\beta}{\pi_3 p_3} r$

FIG. 1

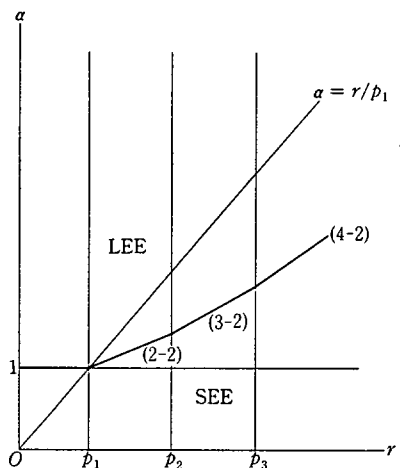


FIG. 2

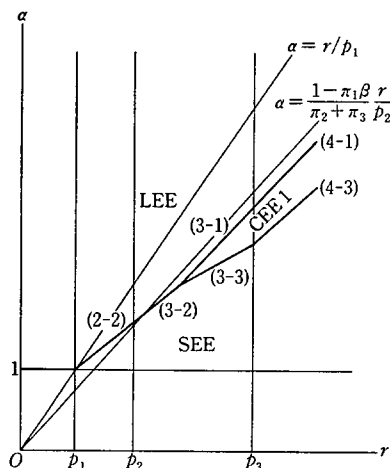


FIG. 3

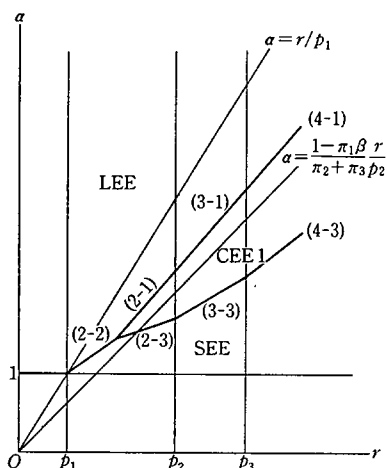
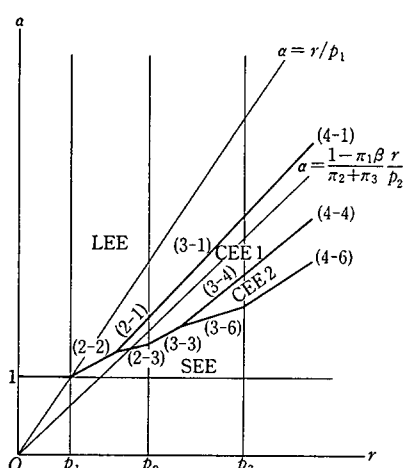


FIG. 4



$\pi_3 p_3$) and summarize the equilibria in (r, α) planes. Figure 1 shows the case where $0 \leq \beta < p_1/E(p)$. We have LEE in the area above the bold line and SEE in the area below it. The two parties are indifferent to LEE and SEE on the boundary.¹⁰ Figure 2 applies to the case where $p_1/E(p) \leq \beta < p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p)$. In this case we have not only LEE and SEE but CEE1 for r sufficiently larger than p_2 . Figure 3 is for the case where $p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p) \leq \beta < p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3)$. This case is different from the previous one in that we have CEE1 even when $p_1 \leq r < p_2$. Figure 4 depicts the case where $p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3) \leq \beta$. In this case we have CEE2 for r sufficiently larger than p_2 . We next as-

¹⁰ Similar notes apply to all the boundaries of different types of equilibrium.

FIG. 5

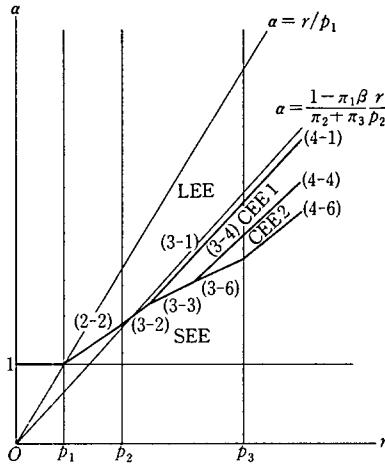


FIG. 6

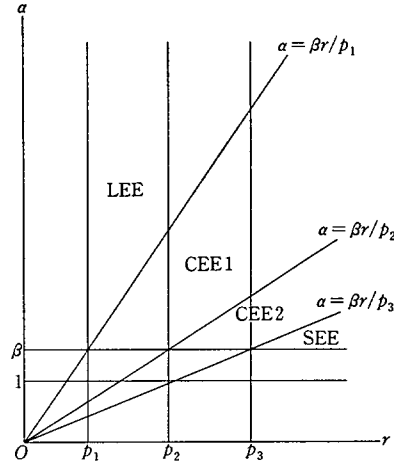


TABLE 2

	$r < p_1$	$p_1 \leq r < p_2$	$p_2 \leq r < p_3$	$p_3 \leq r$
$VL \geq VC1$		$\alpha \geq \beta r/p_1$	$\alpha \geq \beta r/p_1$	$\alpha \geq \beta r/p_1$
$VC1 \geq VC2$			$\alpha \geq \beta r/p_2$	$\alpha \geq \beta r/p_2$
$VC2 \geq VS$				$\alpha \geq \beta r/p_3$

sume that $p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3) \leq p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p)$. The case where $0 \leq \beta < p_1/E(p)$ can be shown by Figure 1 even under this new condition. The case where $p_1/E(p) \leq \beta < p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3)$ has the same figure as Figure 2. The case where $p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3) \leq \beta < p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p)$ is depicted in Figure 5. This differs from Figure 3 in that in this case CEE2 exists, but CEE1 does not exist for any r between p_1 and p_2 . The case where $p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p) \leq \beta$ has the same figure as Figure 4.

We have described the equilibrium for all the cases where $0 \leq \beta < 1$. In the rest of this section we want to discuss briefly the case where $1 \leq \beta < \alpha$. In this case the worker's highest alternative wage becomes equal to βrc . By a procedure similar to what was adopted before, we can easily show that LEE is now $(w, t, x) = (\beta rc, (\beta r - \alpha p_1)c, c)$ and that the maximized expected profit under this condition is given by $V^L = (\alpha E(p) - \beta r)c$. These are slightly different from those obtained around (5). CEE1 is also different. Since $U(x)$ is now increasing in the region of (1b1), it is necessary in CEE1 that $(w - t)/p_1(\alpha - 1) - c/(\alpha - 1) > c$ and $(w - t)/p_2(\alpha - 1) - c/(\alpha - 1) \leq c$. As the worker chooses $x = c$, the condition that $U(x) \geq \beta rc$ implies that $w \geq \beta rc$. Then the maximized expected profit is given by $V^{C1} = \pi_1 v_1 + \pi_2(\alpha p_2 - \beta r)c + \pi_3(\alpha p_3 - \beta r)c$. Though $(w, x) = (\beta rc, c)$ in the equilibrium, the equilibrium security limit is not unique, i.e., $(\beta r - \alpha p_2)c \leq t < (\beta r - \alpha p_1)c$. CEE2 can be obtained similarly. We have the maximized expected profit $V^{C2} = \pi_1 v_1 + \pi_2 v_2 + \pi_3(\alpha p_3 - \beta r)c$ and the equilibrium $(w, x) = (\beta rc, c)$, $(\beta r - \alpha p_3)c \leq t < (\beta r - \alpha p_2)c$. For SEE the firm again has a

continuum of strategies, and the maximized expected profit is the same as (4c). We summarize as before the relative magnitude of the four payoffs in Table 2 and the eventual equilibrium in Figure 6.

IV. Discussion

We want to discuss in this section a few features of our model. Since all the essential results in Arai (1988) also hold in our present model, we would like to emphasize mainly what can be obtained additionally by our new model.

First we can show that the equilibrium arrived at in the previous section maximizes the joint wealth of the initial firm and worker. The method of proof is similar to that in Arai (1988), but the case where $\beta \geq 1$ was not discussed in that paper. We would therefore like to mention it briefly here. When $\beta \geq 1$, the two parties obviously choose $x=c$ and we have the following four types of joint wealth corresponding to the four possible cases:

$$W^L = \alpha E(p)c;$$

$$W^{c1} = \pi_1(v_1 + \beta rc) + \pi_2\alpha p_2c + \pi_3\alpha p_3c;$$

$$W^{c2} = \pi_1(v_1 + \beta rc) + \pi_2(v_2 + \beta rc) + \pi_3\alpha p_3c;$$

$$W^S = \pi_1v_1 + \pi_2v_2 + \pi_3v_3 + \beta rc,$$

where the first expression is the joint wealth in the case where the two parties never separate, the second is the joint wealth in the case where they separate only when $p=p_1$, the third is the joint wealth in the case where they separate only when $p \leq p_2$, and the fourth is the joint wealth in the case where they definitely separate formally. An easy computation shows that we have the same relationships among the above four as those in Table 2 with V^L , V^{c1} , V^{c2} , and V^S replaced by W^L , W^{c1} , W^{c2} , and W^S respectively. Thus the equilibrium is also joint-wealth maximizing when $\beta \geq 1$.

Next we look into the main characteristics of each type of equilibrium. The characteristics of LEE of our present model are almost the same as those described in Arai (1988). One point not mentioned in that paper is that no labor hoarding is necessary when $\beta \geq 1$, since specific capital becomes more useful than general capital even outside the firm. There is nothing especially new about SEE.

Since there are two different kinds of conditional employment equilibrium in our present model, we will now turn our attention to the details of their characteristics. The existence of CEE1 is crucially dependent on the magnitude of β . When $\beta < p_1/E(p)$, CEE1 does not exist (Figure 1). If the productivity of specific capital is low outside the training firm, the firm has to pay the worker a high wage to compensate for the low wage he will receive outside in case of dismissal. The firm finds other types of employment are cheaper than CEE1 in this case. When $\beta \geq p_1/E(p)$, CEE1 exists. As β increases, the area of CEE1 expands, which can be easily seen in Table 1 and Figures 2 through 6 (consider the sequence of Figures 2, 3, 4, and 6, and that of Figures 2, 5, 4, and 6). According to the result above (8), the equilibrium security limit of employment is positive below $\alpha = (1 - \pi_1\beta)r/(\pi_2 + \pi_3)p_2$ and negative above it. In Figures 2 and 5 this critical line lies above $\alpha = \beta r/p_1$.¹¹ Thus

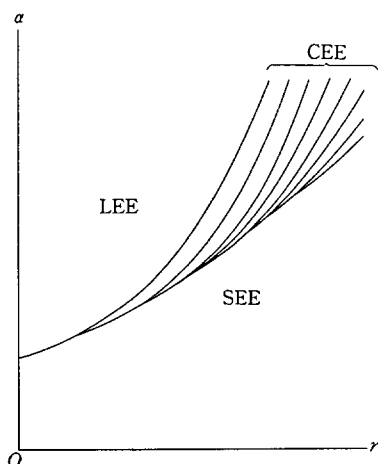
¹¹ $\alpha = \beta r/p_1$ is equivalent to (2-1), (3-1), or (4-1).

CEE1 in these cases brings a loss to the firm whenever $p=p_2$. The firm chooses this strategy because the profit at $p=p_3$ is large. In Figures 3 and 4 there is also labor hoarding, but there can be cases in which the firm has a positive profit when $p=p_2$ since the critical line can lie below $\alpha=\beta r/p_1$. In Figure 6 there is no labor hoarding at $p=p_2$ since the value product is no smaller than the wage at the price. We note that labor hoarding exists even when $r < p_2$ (see Figures 3 and 4). This arises because the wage in CEE1 is larger than rc . The fact that labor hoarding can exist in conditional employment equilibrium is an important result of the present model.

The existence of CEE2 is also crucially dependent on the magnitude of β . The critical value of β for existence, i.e., $\beta=p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3)$ is larger than that of CEE1, i.e., $\beta=p_1/E(p)$. Since the worker will be retained only when $p=p_3$, β must be sufficiently large (the specificity of specific capital must be sufficiently low). As we can see in Figures 4 through 6 and Table 1, CEE2 has similar properties to those of CEE1, but CEE2 never arises when $r < p_2$. Further, there is no labor hoarding in CEE2. This is due to our simplifying assumption of three possible product prices. Though we have assumed that the firm can hire a new worker with c units of general capital when it dismisses the old one, it does not actually hire a new one in CEE1 or CEE2. We can easily see this from the fact that CEE1 arises only when $p_1 \leq r$ and CEE2 arises only when $p_2 \leq r$. If retaining a productive worker is not profitable, hiring a less productive worker is much less profitable.

We have classified the equilibrium in separate figures in terms of the magnitude of β . As β increases, we obtain two sequences of figures that change continuously. One sequence is made up of Figures 1, 2, 3, 4, and 6, while the other is made up of Figures 1, 2, 5, 4, and 6. The former arises when $p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p) < p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3)$, while the latter arises when the inequality is reversed. It is not difficult to show that for given (π_1, π_2, π_3) the former inequality tends to hold when p_3 is close to p_2 and p_1 is small, while the latter (reversed) one tends to hold when p_1 is close to p_2 and p_3 is large.¹² In the former

FIG. 7



¹² The effect of a change in (π_1, π_2, π_3) cannot be described simply, because e.g. an increase in π_1 may or may not lead to a decrease in $\pi_i (i=2, 3)$.

case the advantage of dismissal at p_2 is relatively small because p_2 is close to p_3 , and that of dismissal at p_1 is relatively large because p_1 is small. Thus in Figure 3 CEE2 does not arise, but CEE1 arises even when $r < p_2$. In the latter case the advantage of dismissal at p_2 is relatively large because p_3 is large, and the advantage of dismissal at p_1 is relatively small because p_1 is close to p_2 . Thus in Figure 5 CEE2 arises and the existence of CEE1 is limited to the region where $p_2 \leq r$.

Our analysis has revealed the entire structure of different employment types in the case where the product price takes on three values. The most important feature of the present model is that the results we have obtained so far help us understand what the employment structure will be when the product price takes on many or even a continuum of values. If we classify the structure in terms of the magnitude of β as we have just done, then it is quite natural to conjecture roughly that as β increases, CEE characterized by less complete job security will arise in addition to LEE, SEE and CEE characterized by relatively more complete job security. Though the observation in the previous paragraph implies that the manner in which the structure changes as β increases depends intricately upon the distribution of product price, the approximate structure can be described as in Figure 7. In this more general case, there are as many types of CEE as the number of values the product price takes on minus one. The farther a type of CEE is located from the vertical axis, the less complete is the job security.

V. *Concluding Remarks*

In this paper we have considered how different types of employment characterized by different degrees of employment internalization are determined in a fairly general framework. We have found that the structure of these different types of employment is stratified in a certain fashion as shown in Figures 1 through 7. An advantage of considering the case of three possible product prices has been that it enabled us to conjecture the results of more general cases. As we have emphasized in Arai (1988), each equilibrium reflects the employment system that provides the worker with incentives to accumulate the most desirable combination of specific and general capital in each different condition. In other words, the degree of employment internalization is determined in accordance with the amount of incentives the firm provides to the worker to invest his time and/or effort in specific as opposed to general capital.

APPENDIX

In this appendix we will show which type of equilibrium will arise when $\beta < 1$ by examining which of the four payoffs of V^L , V^{c1} , V^{c2} , and V^S is the largest. To simplify our discussion, we do not pay so much attention to boundaries. Most of the strict inequalities in the following can be replaced by weak ones. These can be easily understood in the process of discussion.

Suppose first that $r < p_1$. Then Table 1 shows that V^L is larger than any other payoff. Thus we have LEE if $r < p_1$. Note that we are assuming that $\alpha > 1$.

Next suppose that $p_1 \leq r < p_2$. Since V^{c1} is unconditionally larger than V^{c2} , we never have CEE2 and we have only to compare V^L , V^{c1} and V^S in this case. We note that equations (2-1), (2-2), and (2-3) intersect in this region of r iff $\beta > p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p)$. When they do not intersect, it can be easily shown that (2-3) lies above (2-2) and (2-2) lies above (2-1). Then we have LEE above (2-2) and SEE below it. When they do intersect, the value of r of the intersection is given by $r = p_1(\pi_2 p_2 + \pi_3 p_3)/(\beta E(p) - \pi_1 p_1)$. When r is between p_1 and this intersection, we have LEE above (2-2) and SEE below it as before. When r is between this intersection and p_2 , (2-1) lies above (2-2) and (2-2) lies above (2-3). Thus we have LEE above (2-1), CEE1 between (2-1) and (2-3), and SEE below (2-3). Note that (9) is satisfied for CEE1, since it is the same as (2-3).

Now suppose that $p_2 \leq r < p_3$. Consider the area below (3-4). By the table we have $V^{c2} > V^{c1}$. Since (3-5) lies above (3-4), $V^{c2} > V^L$ below (3-4). Thus we have neither CEE1 nor LEE below (3-4). If $\beta \leq p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3)$, (3-6) lies above (3-4) and we have SEE below (3-4). If $\beta > p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3)$, (3-6) and (3-4) intersect and we have CEE2 in the area below (3-4) and above (3-6). Note that (11) is satisfied for CEE2, since it is the same as (3-6). Obviously we have SEE in the area where CEE2 does not prevail. Consider next the area above (3-4). If $\beta < p_1/E(p)$, (3-3) lies above (3-2) and (3-2) lies above (3-1). In this case we have LEE above (3-2) and SEE below it. If $\beta \geq p_1/E(p)$, the value of (3-1) is larger than that of (3-3) at $r = p_3$. Thus CEE1 prevails in the area between (3-1) and (3-3). We note that (9) holds in this area, because (3-3) lies above (9). If $\beta \leq p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p)$, (3-1) and (3-3) intersect at a value of r between p_2 and p_3 . Between p_2 and this value of r , we have LEE above (3-2) and SEE below it. Between this value of r and p_3 , we have LEE above (3-1) and SEE below (3-3). If $\beta > p_1(\pi_1 + \pi_2 + \pi_3 p_3/p_2)/E(p)$, (3-1) and (3-3) do not intersect between p_2 and p_3 . In this case we have LEE above (3-1) and SEE below (3-3).

Finally suppose that $p_3 \leq r$. We can easily show that (4-6) lies above (4-4) if $\beta < p_1/E(p)$. Since (4-5) lies above (4-4), we have SEE below (4-4) if $\beta < p_1/E(p)$. Further, if $\beta < p_1/E(p)$, (4-3) lies above (4-2) and (4-2) lies above (4-1). Thus we have LEE above (4-2) and SEE below it. On the other hand, if $p_1/E(p) \leq \beta < p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3)$, (4-1) lies above (4-2) and (4-2) lies above (4-3), though (4-6) and (4-5) lie above (4-4) as before. Thus we have LEE above (4-1), CEE1 between (4-1) and (4-3), and SEE below (4-3). Note that (9) is satisfied for CEE1 since (4-3) lies above (9). If $\beta \geq p_2/(\pi_1 p_2 + \pi_2 p_2 + \pi_3 p_3)$, (4-5) lies above (4-4) and (4-4) lies above (4-6). Further (4-1) lies above (4-4) and (4-4) lies above (4-3). Thus we have LEE above (4-1), CEE1 between (4-1) and (4-4), CEE2 between (4-4) and (4-6), and SEE below (4-6). It is easy to show that (9) and (11) are satisfied for CEE1 and CEE2 respectively.

So far we have assumed that $\alpha > 1$. If $\alpha \leq 1$, it is obvious that the firm chooses a strategy that leads to $x=0$. There are a continuum of such strategies (see Note 6). Thus if $\alpha \leq 1$, SEE arises.

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